Q1a

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

a) $y = e^t \Rightarrow \frac{dy}{dt} = e^t$

$x = \sin 2t \Rightarrow \frac{dx}{dt} = 2\cos 2t$

$$\boxed{\frac{dy}{dx} = \frac{e^t}{2\cos 2t}} = \frac{1}{2}e^t \sec 2t$$

Q1b

equations

[3]

ıt (0 , 1) and find the

[2]

From part (a), $\frac{dy}{dx} = \frac{e^t}{2\cos 2t}$

b) At the point $(0, 1)$, $x = 0$ and $y = 1$

If $y = 1$: $e^t = 1 \Rightarrow t = \ln(1) = 0$

If $t = 0$: $x = \sin(0) = 0$

Therefore the graph passes through the point $(0, 1)$ when $t = 0$

The gradient at $(0, 1)$ is

$$\frac{dy}{dx} = \frac{e^0}{2\cos(0)}$$

Substitute $t = 0$ into the result from part (a)

$$\frac{dy}{dx} = \frac{1}{2}$$

Q2a

a) $x = 8t - 4 \Rightarrow 8t = x + 4 \Rightarrow 4t = \frac{x+4}{2}$

$$y = 16t^2 - 16t + 5$$
$$= (4t)^2 - 4(4t) + 5$$
$$= \left(\frac{x+4}{2}\right)^2 - 4\left(\frac{x+4}{2}\right) + 5$$
$$= \frac{x^2 + 8x + 16}{4} - 2x - 8 + 5$$
$$= \frac{x^2}{4} + 2x + 4 - 2x - 8 + 5$$

$$\boxed{y = \frac{1}{4}x^2 + 1}$$

The domain is the range of $x = 8t - 4$

$0 \le t \le 1 \Rightarrow 0 \le 8t \le 8 \Rightarrow -4 \le 8t - 4 \le 4$

The domain of $f(x)$ is $-4 \le x \le 4$

Q2b

From part (a), $y = \frac{1}{4}x^2 + 1$, $-4 \le x \le 4$

b) The minimum height is 1 m (when $x = 0$)

When $x = -4$, $y = \frac{1}{4}(-4)^2 + 1 = 5$

When $x = 4$, $y = \frac{1}{4}(4)^2 + 1 = 5$

So the maximum height is 5 m

The difference between the maximum and minimum heights is 4 m.

Q2C

From part (a), $y = \frac{1}{4}x^2 + 1$, $-4 \le x \le 4$

c) At point A, $x = -4$, therefore the wall is located at $x = 3$.

When $x = 3$,

$$y = \frac{1}{4}(3)^2 + 1 = \frac{13}{4} = 3.25$$

$\boxed{\text{The wrecking ball will strike the wall at a height of } 3.25 \text{ m}}$

Q3a

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

a) $y = 5\sin\theta \Rightarrow \frac{dy}{d\theta} = 5\cos\theta$

$x = 2\cos 3\theta \Rightarrow \frac{dx}{d\theta} = -6\sin 3\theta$

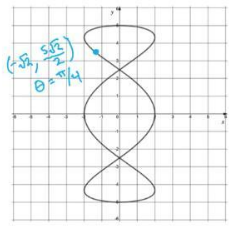
$$\boxed{\frac{dy}{dx} = -\frac{5\cos\theta}{6\sin 3\theta}}$$

Note: Because the arguments of cos and sin are different here (θ and 3θ respectively), we can't use $\frac{\cos}{\sin} = \cot$ here!

Q3b

The graph of the curve C shown below is defined by the parametric equations

$x = 2\cos 3\theta \qquad y = 5\sin\theta \qquad 0 \le \theta \le 2\pi$



(a) Find an expression for $\frac{dy}{dx}$ in terms of θ.

(b) (i) Show that the gradient of the tangent to C, at the point where $\theta = \frac{\pi}{4}$, is $-\frac{5}{6}$.

(ii) Hence find the equation of the tangent to C at the point where $\theta = \frac{\pi}{4}$.

[4]

Equation of a line with gradient m through (x_1, y_1) is

$y - y_1 = m(x - x_1)$

From part (a), $\frac{dy}{dx} = -\frac{5\cos\theta}{6\sin 3\theta}$

b) (i) When $\theta = \pi/4$, the gradient of the tangent is

$$\boxed{\frac{dy}{dx} = -\frac{5\cos(\pi/4)}{6\sin(3\pi/4)} = -\frac{5(\sqrt{2}/2)}{6(\sqrt{2}/2)} = -\frac{5}{6}}$$

(ii) When $\theta = \pi/4$,

$x = 2\cos(3\pi/4) = 2(-\sqrt{2}/2) = -\sqrt{2}$

$y = 5\sin(\pi/4) = 5(\sqrt{2}/2) = \frac{5\sqrt{2}}{2}$

So the equation of the gradient is

$$y - \frac{5\sqrt{2}}{2} = -\frac{5}{6}(x - (-\sqrt{2}))$$

$$y - \frac{5\sqrt{2}}{2} = -\frac{5}{6}x - \frac{5\sqrt{2}}{6}$$

$$y = -\frac{5}{6}x - \frac{5\sqrt{2}}{6} + \frac{5\sqrt{2}}{2}$$

$$\boxed{y = -\frac{5}{6}x + \frac{5\sqrt{2}}{3}}$$

This could also be given in the form

$$5x + 6y - 10\sqrt{2} = 0$$

Q4a

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

a) $x = \frac{1}{t^2} = t^{-2} \Rightarrow \frac{dx}{dt} = -2t^{-3} = -\frac{2}{t^3}$

$y = t + \frac{1}{t} = t + t^{-1} \Rightarrow \frac{dy}{dt} = 1 - t^{-2} = 1 - \frac{1}{t^2} = \frac{t^2 - 1}{t^2}$

$$\frac{dy}{dx} = \frac{t^2 - 1/t^2}{-2/t^3}$$

$$\boxed{\frac{dy}{dx} = -\frac{t(t^2 - 1)}{2}}$$

Q4b

The curve C has parametric equations

$$x = \frac{1}{t^2} \qquad y = t + \frac{1}{t} \qquad t > 0$$

(a) Find an expression, in terms of t, for $\frac{dy}{dx}$. [3]

(b) (i) Find the gradient of the tangent to C at the point where $t = \frac{1}{2}$.

(ii) Hence find the equation of the normal to C at the point where $t = \frac{1}{2}$. [5]

From part (a), $\frac{dy}{dx} = -\frac{t(t^2-1)}{2}$

gradient of normal $= -\frac{1}{dy/dx}$

Equation of a line with gradient m through (x_1, y_1) is $y - y_1 = m(x - x_1)$

b) (i) The gradient of the tangent is

$$\frac{dy}{dx} = -\frac{(\frac{1}{2})((\frac{1}{2})^2-1)}{2} = -\frac{(\frac{1}{2})(-\frac{3}{4})}{2} = \boxed{\frac{3}{16}}$$

(ii) The gradient of the normal is $-\frac{1}{3/16} = -\frac{16}{3}$

when $t = \frac{1}{2}$,

$$x = \frac{1}{(1/2)^2} = 4$$

$$y = \frac{1}{2} + \frac{1}{1/2} = \frac{5}{2}$$

The equation of the normal is

$$y - \frac{5}{2} = -\frac{16}{3}(x - 4)$$

$$y - \frac{5}{2} = -\frac{16}{3}x + \frac{64}{3}$$

$$\boxed{y = -\frac{16}{3}x + \frac{143}{6}}$$

This equation could also be given in the form

$$32x + 6y - 143 = 0$$

Q5a

The curve C has parametric equations

$$x = t^2 - 4 \qquad y = 3t$$

(a) Show that at the point $(0, 6)$, $t = 2$ and find the value of $\frac{dy}{dx}$ at this point. [4]

(b) The tangent at the point $(0, 6)$ is parallel to the normal at the point P.
Find the exact coordinates of point P

a) At the point $(0, 6)$, $x = 0$ and $y = 6$

$\underline{x = 0}$: $t^2 - 4 = 0 \Rightarrow t^2 = 4 \Rightarrow t = \underline{2}$ or -2

$\underline{y = 6}$: $3t = 6 \Rightarrow t = \underline{2}$

So at the point $(0, 6)$, $t = 2$

The value of t must work for x $\underline{\text{and}}$ y

$$y = 3t \Rightarrow \frac{dy}{dt} = 3$$

$$x = t^2 - 4 \Rightarrow \frac{dx}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{3}{2t}$$

So at $(0, 6)$ with $t = 2$

$$\boxed{\frac{dy}{dx} = \frac{3}{4}}$$

Q5b

The curve C has parametric equations

$$x = t^2 - 4 \qquad y = 3t$$

(a) Show that at the point $(0, 6)$, $t = 2$ and find the value of $\frac{dy}{dx}$ at this point.

[4]

(b) The tangent at the point $(0, 6)$ is parallel to the normal at the point P.
Find the exact coordinates of point P

[3]

From part (a),

$\frac{dy}{dx} = \frac{3}{2t}$ in general

$\frac{dy}{dx} = \frac{3}{4}$ at the point $(0, 6)$

This is the gradient of the tangent at $(0, 6)$

gradient of normal $= -\frac{1}{dy/dx}$

b) At point P, the normal has gradient $3/4$

The gradient of the normal is given by

$$-\frac{1}{dy/dx} = -\frac{1}{3/2t} = -\frac{2t}{3}$$

At point P,

$$-\frac{2t}{3} = \frac{3}{4} \implies 8t = -9 \implies t = -\frac{9}{8}$$

When $t = -\frac{9}{8}$,

$$x = \left(-\frac{9}{8}\right)^2 - 4 = \frac{81}{64} - 4 = -\frac{175}{64}$$

$$y = 3\left(-\frac{9}{8}\right) = -\frac{27}{8}$$

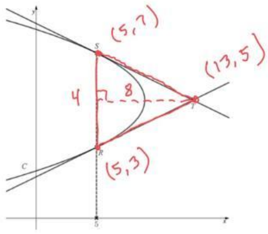
Point P has coordinates $\left(-\frac{175}{64}, -\frac{27}{8}\right)$

Q6

A curve C has parametric equations

$$x = 9 - t^2 \qquad y = 5 - t$$

The tangents to C at the points R and S meet at the point T, as shown in the diagram below.



Given that the x-coordinate of both points R and S is 5, find the area of the triangle RST.

[10]

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Equation of a line with gradient m through (x_1, y_1) is $y - y_1 = m(x - x_1)$

When $x = 5$,

$$9 - t^2 = 5 \implies t^2 = 4 \implies t = 2 \text{ or } -2$$

If $t = 2$, $y = 5 - 2 = 3$

If $t = -2$, $y = 5 - (-2) = 7$

So point R is $(5, 3)$ and point S is $(5, 7)$

$$x = 9 - t^2 \implies \frac{dx}{dt} = -2t$$

$$y = 5 - t \implies \frac{dy}{dt} = -1$$

$$\frac{dy}{dx} = \frac{-1}{-2t} = \frac{1}{2t}$$

Find dy/dx in terms of t

Therefore at point R with $t = 2$, the gradient of the tangent is $\frac{1}{2(2)} = \frac{1}{4}$

So the equation of the tangent is

$$y - 3 = \frac{1}{4}(x - 5) \implies y = \frac{1}{4}x + \frac{7}{4}$$

And at point S with $t = -2$, the gradient of the tangent line is $\frac{1}{2(-2)} = -\frac{1}{4}$

So the equation of the tangent is

$$y - 7 = -\frac{1}{4}(x - 5) \implies y = -\frac{1}{4}x + \frac{33}{4}$$